Collaboration Projects

Analysis and Evaluation of Spectral Efficiency in Multihop Transmission

We analyze and evaluate spatial wireless spectral efficiency, when adaptive rate control is applied to multihop transmission where wireless nodes located between transmitting and receiving nodes act as relaying nodes, and clarified its effects. This research was conducted jointly with the Yoshida laboratory (Professor Susumu Yoshida), Graduate School of Informatics, Kyoto University.

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1. Introduction

In preparation for rolling out various services making use of wireless transmission, issues such as “expanding coverage” and “increasing spectral efficiency” are very important, considering the limited availability of frequency resources.

In wireless transmission in general, it is not possible to assume that the transmission power output of a transmitting node can be increased without limit due to restrictions posed by the equipment itself as well as various regulations. Similarly, the distance at which communication is possible cannot be extended indefinitely, either. On the other hand, even at distances between transmitting and receiving nodes where communication is possible, the available end-to-end bit rate in a communication system is roughly proportional to the received power, which means that the end-to-end bit rate decreases as the distance between the transmitting and receiving nodes increases. In order to solve these issues, “multihop transmission,” where transmission is conducted via relaying nodes positioned between the transmitting and receiving nodes, is a useful way to expand the range at which communication is possible and to improve the communication bit rate, and the effects of adopting this approach in communication systems have been clarified [1] [2].

However, in wireless communication systems, using multihop transmission runs the risk of reducing the overall system capacity because wireless resources such as frequency bands, spread code and time slots are consumed in relaying process. Moreover, losses in terms of multiple-access control are also a concern because the number of transmitting nodes is greater in multihop transmission compared to single-hop transmission. However, assuming that the access control operates in an ideal manner (i.e., the losses due to multiple access are zero), this study uses mathematical models to analyze the impacts of multihop transmission on system capacity and demonstrates that multihop transmission can significantly contribute to expansion of the system capacity.

2. Bandwidth Efficiency of Multihop Transmission

2.1 Adaptive Modulation in Single-Hop Transmission

The purpose of this research is to clarify the spectral efficiency of wireless communication using multihop transmission. We thus use “bandwidth efficiency,” which is the maximum end-to-end bit rate normalized by bandwidth, as the index of the evaluation. The actual end-to-end bit rate for a given system can then be obtained by multiplying the bandwidth efficiency by the bandwidth to be used. This section explains how to obtain the bandwidth efficiency of the single-hop transmission shown in **Figure 1** (a) in order to clarify the bandwidth efficiency of multihop transmission.

In “single-hop transmission,” where a transmitting node and a receiving node communicate directly, the upper limit of the bandwidth efficiency is defined explicitly by Shannon’s theorem. If the Signal to Noise Ratio (SNR) at a receiving node is given, the bandwidth efficiency f is expressed by equation (1).

$$f=\log_2(1+\mathrm{SNR}) \qquad (1)$$

This equation indicates the absolute theoretical limit, i.e., “transmission at a higher bit rate is impossible,” and bit rates

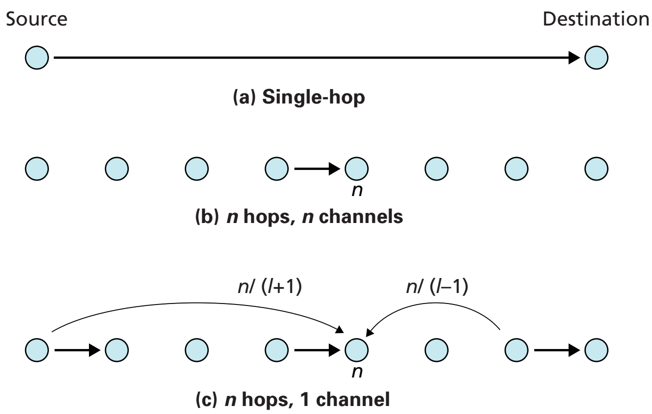

Figure 1 Multihop transmission model

used in actual wireless transmission are necessarily lower than the value obtained from this equation. Instead, the following approximated expression, which is derived from applying Quadrature Phase Shift Keying (QPSK) to 64 Quadrature Amplitude Modulation (64QAM), is used as an end-to-end bit rate model that matches the reality better in case of adaptive modulation. This model approximates the bandwidth efficiency f_1 of single-hop as shown in equation (2).

$$f_1(\gamma, p_b)=\log_2(1+\beta_1\gamma) \qquad (2)$$
$$\beta_1=-1.5/\ln(5p_b)$$

Where p_b is the required Bit Error Rate (BER) and γ is the received Carrier to Interference and Noise Ratio (CINR).

Since this model cannot be used for transmission rates below QPSK, we extend the model by assuming that the same symbol is repeatedly transmitted. From the approximation above, the single-hop bandwidth efficiency f_1 can be expressed by equation (3) [3].

$$f_1(\gamma, p_b)=\begin{cases} 2\beta_1\gamma/3 & 3/4\beta_1\leq\gamma<3/\beta_1 \\ \log_2(1+\beta_1\gamma) & 3/\beta_1\leq\gamma\leq 63/\beta_1 \end{cases} \qquad (3)$$

2.2 Multihop Transmission without Spatial Channel Reuse

We made the following assumptions in order to examine the bandwidth efficiency of multihop transmission. Wireless relaying nodes are placed between the transmitting node and the receiving node at equal intervals (Fig. 1 (b)). Spatial channel reuse is not performed. This means that transmission/reception using the same frequency at the same time is not conducted regardless of the distance between the individual hops in the multihop transmission. That is, only one node can be either transmitting or receiving at a given time or frequency. Note that it has no influence on the essential arguments, whether time- or frequency-division is used.

The per-hop distance in n-hop transmission is 1/n of the distance in the corresponding single-hop transmission, which means that the received CINR is n^{α} times larger than for the single-hop transmission, where α is the propagation attenuation constant. Moreover, the per-hop BER must be larger than p_b/n in order to achieve the same BER as the single-hop transmission at the source and destination endpoints of the n-hop transmission. Furthermore, if spatial frequency reuse is not performed in the transmission at every hop, the bandwidth efficiency becomes n times the per-hop bandwidth. By combining these factors, the bandwidth efficiency f_n of the n-hop transmission can be expressed as in equation (4), using the bandwidth efficiency f_1 of the corresponding single-hop transmission.

$$f_n(\gamma, p_b)=\frac{1}{n}f_1(\gamma n^{\alpha}, p_b/n) \qquad (4)$$

This relationship can be expressed graphically as shown in **Figure 2**, where the horizontal axis indicates the received CINR at a destination node and the vertical axis indicates the bandwidth efficiency. From this figure, it can be concluded that no gain can be obtained by adopting multihop transmission if the received CINR at single-hop transmission is 12 dB or larger. This also shows that the number of hops yielding the maximum bandwidth efficiency can be determined uniquely in other cases as well. For example, if the received CINR is around 4 dB, the bandwidth efficiency can be significantly improved by

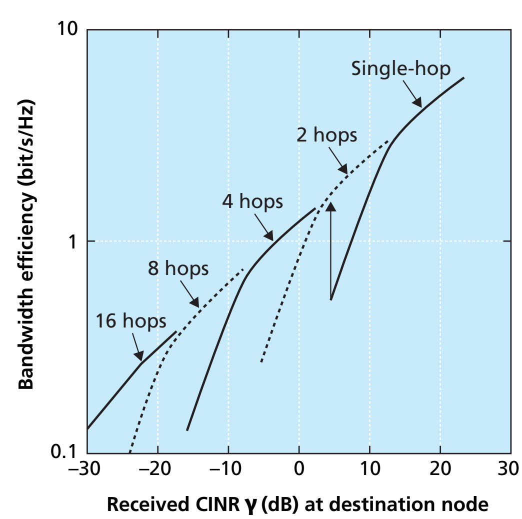

Figure 2 Bandwidth efficiency in multihop transmission

using 2-hop transmission rather than single-hop transmission (as indicated by the arrow pointing to the top in the figure).

The bandwidth efficiency of multihop transmission discussed above is determined by two factors, the received CINR at each hop and the required BER. **Figure 3** shows the impact of error accumulation on the bandwidth efficiency assuming multihop transmission. This also shows the upper limit on the bandwidth efficiency determined by the Shannon capacity. As can be seen from the figure, the CINR is not influenced significantly even if the required BER varies. In other words, the main factor that can improve the bandwidth efficiency of multihop transmission is changes of received CINR.

2.3 Multihop Transmission with Spatial Channel Reuse

Here we assume that n-hop transmission is performed using l orthogonal channels (Fig. 1 (c)). This means that transmission is conducted at the same time at l-hop intervals assuming that each hop is transmitted using time division (the same frequency is used at l-hop intervals in case of frequency division).

According to this assumption, co-channel interference occurs in transmission at a certain hop due to transmission l hops forward, $2l$ hops forward and so forth. Taking the influence of co-channel interference into consideration, the bandwidth efficiency is calculated as in equations (5) and (6).

$$\frac{C_{n,l}}{I_{n,l}}=\left\{\sum_{i=1}^{\lceil n/l\rceil-1}[\lceil i/2\rceil l+(-1)^{i}]^{-\alpha}\right\}^{-1}\equiv\xi \quad (5)$$

$$f_{n,l}(\gamma, p_b)=\frac{1}{l}f_1((1/\gamma n^{\alpha}+1/\xi)^{-1}, p_b/n) \quad (6)$$

Where the per-hop carrier wave power is denoted $C_{n,l}$ and the interference power of the link that receives the greatest interference is denoted $I_{n,l}$.

The value of l that maximizes the bandwidth efficiency can be obtained for a given number of hops n. **Figure 4** shows the bandwidth efficiency when this l is used. Since no significant change due to frequency reuse is seen in the bandwidth efficiency in case of 4 hops or less, the figure only shows bandwidth efficiency for 8 hops and 16 hops for the case where frequency reuse is adopted. From this figure, it is possible to improve the bandwidth efficiency via spatial frequency reuse in case the received CINR is –7 dB or less, which implies that the number of hops that maximizes the bandwidth efficiency is 8 or larger.

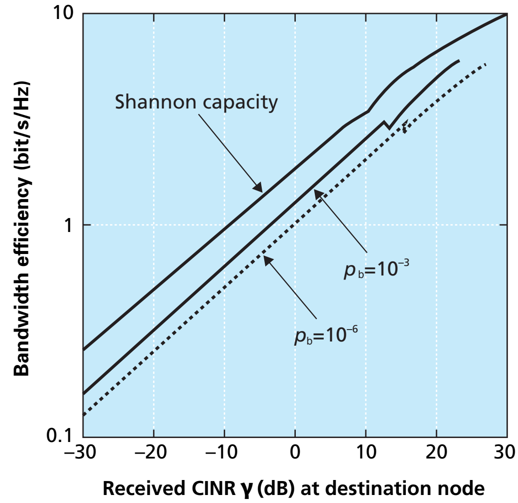

Figure 3 Impact of error accumulation on bandwidth efficiency

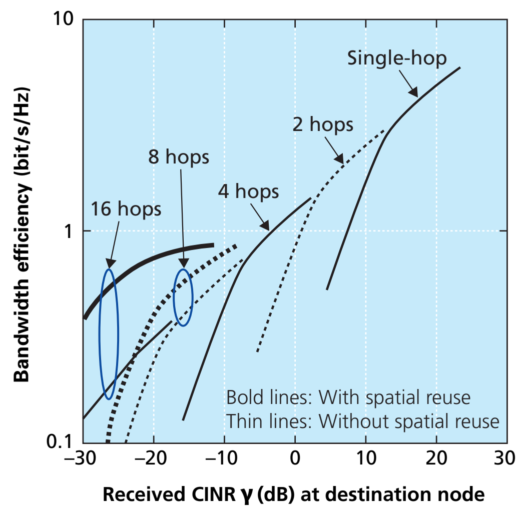

Figure 4 Bandwidth efficiency with spatial channel reuse

2.4 Spatial Spectral Efficiency in Mesh Networks

So far, we have only discussed cases where the end-to-end bit rate is limited by the received power to thermal noise ratio or received power to "thermal noise and interference power among hops" ratio. According to the results found above, it is expected that the received CINR in each hop can be improved in multihop transmission compared to single-hop transmission. This section investigates how the spatial spectral efficiency changes by multihop transmission in multi-user environments.

The bandwidth efficiency is expressed by equation (7).

$$f_n=(\gamma_{QPSK}/z^{\alpha}, p_b)=T \quad (7)$$

Where the frequency reuse distance is denoted z and the bandwidth efficiency is maintained at T in comparison with single-hop QPSK transmission (the spatial spectral efficiency is

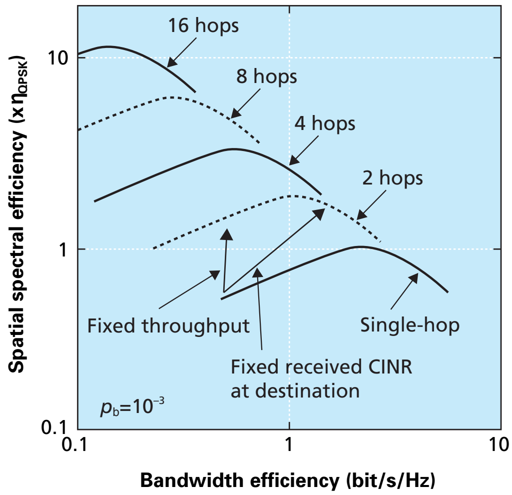

Figure 5 Characteristics of spatial spectral efficiency in multihop transmission

denoted η_{QPSK} and the bandwidth efficiency T).

Based on this, the spatial spectral efficiency in multihop transmission may be expressed as in equation (8).

$$\eta_s=\begin{cases}\frac{n^2T}{2}\left(\frac{2}{nT}\frac{\beta_n}{\beta_1}\right)^{2/\alpha}\eta_{QPSK} & \frac{1}{2_n}\leq T<\frac{2}{n}\\ \frac{n^2T}{2}\left(\frac{3}{2^{nT}-1}\frac{\beta_n}{\beta_1}\right)^{2/\alpha}\eta_{QPSK} & \frac{2}{n}\leq T\leq\frac{6}{n}\end{cases} \quad (8)$$

$$\beta_n=-1.5/\ln(5p_b/n)$$

Figure 5 shows the characteristics when α=3.5, which is often used for propagation models in urban areas. The horizontal axis indicates the per-hop bandwidth efficiency and the vertical axis indicates the spatial spectral efficiency. This shows the characteristics when the modulation method is QPSK; a maximum point is observed in the characteristics for each number of hops. As seen in the figure, in areas where the spectral efficiency is low (areas with low CINR, i.e., areas where the frequency reuse distance is short), the spatial spectral efficiency can be improved by more than a factor of 10 by adopting multihop transmission.

3. Conclusion

We analyzed the effects of multihop transmission on the bandwidth efficiency and spatial spectral efficiency in wireless transmission with adaptive rate control using mathematical models, and found that a significant improvement of the spatial spectral efficiency can be achieved through such schemes. This, for example, shows that the system capacity of certain cellular systems can be improved by a factor of 10 or more. To make use of this research results in the future, we will investigate the impact of the idealized access control assumed, by considering realistic access control schemes in analysis.

References

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[2] T. Mukai, E. Murata and S. Yoshida: “Study on Channel Selection Algorithm and Number of Established Routes of Multi-hop Autonomous Distributed Radio Networks,” IEICE Trans. Com (Japanese Edition), Vol. J85-B, No. 12, Dec. 2002.

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Abbreviations

64QAM: 64 Quadrature Amplitude Modulation
BER: Bit Error Rate
CINR: Carrier to Interference and Noise Ratio
QPSK: Quadrature Phase Shift Keying
SNR: Signal to Noise Ratio